

Scattering phase shift and resonance properties on the lattice: an introduction

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Abstract

We describe the method for extracting the elastic scattering phase shift from a lattice simulation at an introductory level, for non-lattice practitioners. We consider the scattering in a resonant channel, where the resulting phase shift $\delta(s)$ allows the lattice determination of the mass and the width of the resonance from a Breit-Wigner type fit. We present the method for the example of P-wave $\pi\pi$ scattering in the ρ meson channel.

1 Introduction

The determination of the strong decay width of a hadronic resonance in lattice QCD is a much more demanding task than the determination of its approximate mass. The only available method (that was applied up to now) was proposed by Lüscher [1] and is rather indirect. It applies for the case when the resonance appears in the elastic scattering of two hadrons $H_1 H_2 \rightarrow R \rightarrow H_1 H_2$.

- First, the energy spectrum E_n of the system of two interacting hadrons $H_1 H_2$ enclosed in a few-fermi box has to be determined. The system is illustrated in Fig. 1. The spectrum in a finite box E_n is discrete and few (one or two) lowest energy levels have to be determined by lattice simulation.
- The shift of the energy E_n with respect to the non-interacting energy $E_{H_1}(\mathbf{p}_1) + E_{H_2}(\mathbf{p}_2)$ ($E_{H_i}(\mathbf{p}_i) = \sqrt{m_i^2 + \mathbf{p}_i^2}$) gives info on the interaction between H_1 and H_2 . Lüscher derived a rigorous relation between the energy shift $E_n - E_{H_1} - E_{H_2}$ and the elastic phase shift $\delta(s)$ for $H_1 H_2$ scattering in continuum [1]. The measured energies E_n can be used to extract the phase shift $\delta(s)$ evaluated at $s = E_n^2 - \mathbf{P}^2$, where E_n is

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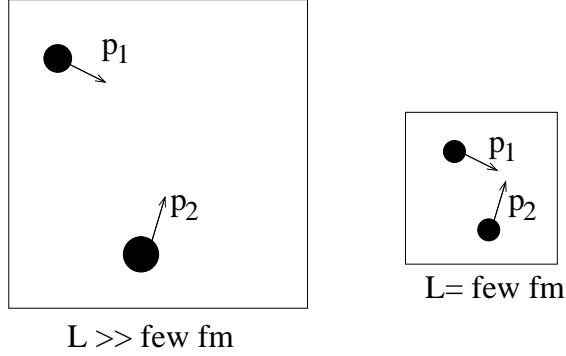


Figure 1: The energy of two hadrons in a box of size L . On the left, $L \gg \text{fm}$ and $E(L) \simeq E_{H_1}(\mathbf{p}_1) + E_{H_2}(\mathbf{p}_2)$. On the right, $L \simeq \text{few fm}$ and energy gets shifted due to their interaction, i.e. $E(L) \simeq E_{H_1}(\mathbf{p}_1) + E_{H_2}(\mathbf{p}_2) + \Delta E(L)$.

the energy of the system and \mathbf{P} its total momentum. In order to extract $\delta(s)$ at several different values of s , the simulations are done for several choices of total momenta \mathbf{P} of the $H_1 H_2$ system, which leads to different values of $s = E_n^2 - \mathbf{P}^2$.

- The resulting dependence of $\delta(s)$ as a function of s can be used to extract the mass m_R and the width Γ_R of the resonance R , which appears in the elastic channel $H_1 H_2 \rightarrow R \rightarrow H_1 H_2$. For this purpose, the $\delta(s)$ can be fitted with a Breit-Wigner form or some other phenomenologically inspired form, which depend on m_R and Γ_R .

The described method, needed for the determination of the resonance width Γ_R , is rather challenging. It requires very accurate determination of a few lowest energy levels of the system $H_1 H_2$, since the resulting phase shift depends ultimately on the energy shift. Among all the meson resonances, this method has been up to now rigorously applied only to ρ resonance. Although Lüscher proposed the method already in late 80's [1], the first lattice attempt to employ it to hadronic resonances had to wait until 2007 [2]. Since then, several studies of ρ have been carried out [3, 4], with the most up to date ones [5, 6, 7].

This talk briefly describes the method to extract $\delta(s)$, m_R and Γ_R on an example of $\pi\pi$ scattering in the ρ channel. It is based on a recent simulation [6], which is the statistically most accurate determination of any strong meson width on one lattice ensemble. The purpose of this talk is to highlight the main physical reasoning, which lies behind the lattice extraction of $\delta(s)$, m_R and Γ_R , omitting most of technical details.

The sections follow the order of steps required, which are listed as items in the introduction. Section II describes the determination of spectrum E_n of the coupled system $H_1 H_2 \leftrightarrow R$. The Section III described why E_n allow one to extract the elastic phase shift $\delta(s)$. The extraction of the resonance parameters m_R and Γ_R from the phase shift $\delta(s)$ is done in Section IV. We end with conclusions.

2 Spectrum of two hadrons in a finite box

The ρ meson is a resonance in $\pi\pi$ scattering in P -wave, and has quantum numbers $I^G(J^{PC}) = 1^+(1^{--})$. The total momentum \mathbf{P} of the coupled $\pi\pi - \rho$ system can have values $\frac{2\pi}{N_L}\mathbf{d}$, $d \in Z^3$ due to the periodic boundary condition in the spatial direction, and we use the following three choices

$$\mathbf{P} = (0, 0, 0) , \frac{2\pi}{N_L}(0, 0, 1) , \frac{2\pi}{N_L}(1, 1, 0) \quad \text{and permutations} . \quad (1)$$

This enables us to obtain several values of $s = E_n^2 - \mathbf{P}^2$ for the system, thereby allowing the determination of $\delta(s)$ for these values of s without changing the spatial volume.

Our simulation is performed on an ensemble of 280 [8] gauge configurations with dynamical u/d quarks, where the valence and dynamical quarks employ improved Wilson-Clover action. The corresponding pion mass is $m_\pi a = 0.1673 \pm 0.0016$ or $m_\pi = 266 \pm 4$ MeV. The lattice spacing is $a = 0.1239 \pm 0.0013$ fm and we employ a rather small volume $N_L^3 \times N_T = 16^3 \times 32$, which allows us to use the costly full distillation method [9] for evaluating the quark contractions.

On the lattice, the discrete energies of the system E_n can be extracted after computing the dependence of the correlation matrix $C_{ij}(t_f, t_i)$ on Euclidean time $t_f - t_i$

$$C_{ij}(t_f, t_i) = \langle 0 | \mathcal{O}_i(t_f) \mathcal{O}_j^\dagger(t_i) | 0 \rangle = \sum_n \langle \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j^\dagger \rangle e^{-E_n(t_f - t_i)} . \quad (2)$$

The analytical expression on the right is obtained by inserting the complete set $\sum_n |n\rangle\langle n|$ of physical states n with given quantum numbers. The interpolators \mathcal{O}_i have the quantum numbers of the system in question. In our case the interpolators have quantum numbers $J^{PC} = 1^{--}$ and $|I, I_3\rangle = |1, 0\rangle$ and total three-momentum \mathbf{P} . They have to couple well to the $\pi\pi$ state and the quark-antiquark resonance ρ .

For each choice of \mathbf{P} (1), we use 16 interpolators, listed in detail in Eq. (21) of [6]. We employ fifteen interpolators of quark-antiquark type

$$\mathcal{O}_i^{\bar{q}q}(t) = \sum_{\mathbf{x}} e^{i\mathbf{P}\mathbf{x}} \frac{1}{\sqrt{2}} [\bar{u}\mathcal{F}_i u(t, \mathbf{x}) + \bar{d}\mathcal{F}_i d(t, \mathbf{x})] , \quad (3)$$

where \mathcal{F}_i denotes different color-spin-space structures with the same resulting quantum number $J^{PC} = 1^{--}$ and $|I, I_3\rangle = |1, 0\rangle$. We use also one $\pi(\mathbf{p}_1)\pi(\mathbf{p}_2)$ interpolator, where each pion is projected to a definite momentum

$$\mathcal{O}^{\pi\pi}(t) = \frac{1}{\sqrt{2}} [\pi^+(\mathbf{p}_1)\pi^-(\mathbf{p}_2) - \pi^-(\mathbf{p}_1)\pi^+(\mathbf{p}_2)] , \quad \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P} \quad \pi^\pm(\mathbf{p}_i) = \sum_{\mathbf{x}} e^{i\mathbf{p}_i\mathbf{x}} \bar{q}\gamma_5\tau^\pm q(t, \mathbf{x}) \quad (4)$$

In practice, the $\pi\pi$ interpolator is the most important among our 16 interpolators, since it couples to the scattering state much better than the quark-antiquark interpolators. Let us note that all other lattice studies aimed at Γ_ρ used at most one quark-antiquark and one $\pi\pi$

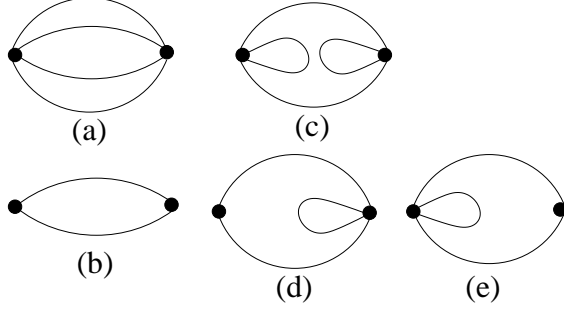


Figure 2: Contractions for $I = 1$ correlators with $\bar{q}q$ (3) and $\pi\pi$ (4) interpolators.

interpolator, which may not always allow for reliable extraction of the first excited energy level E_2 .

Given the 16 interpolators, we compute the 16×16 correlation matrix $C_{ij}(t_f, t_i)$ for all initial and final time-slices $t_i, t_f = 1, \dots, N_T = 32$. The needed Wick contractions that enter the correlation matrix with our $\bar{q}q$ and $\pi\pi$ interpolators are depicted in Fig. 2. The contributions (a,c,e) in Fig. 2 cannot be evaluated solely from the quark propagator from one point (t_i, \mathbf{x}_i) to all other points of the lattice (such a propagator allowed most of the spectroscopy studies in the past). The contributions (a,c,e) require the propagators from all and to all points on the lattice, which is too costly to evaluate in practice. We use the recently proposed distillation method for this purpose [9], which enables the exact computation of the required contractions.

We average the resulting correlators (i) over all initial time slices t_i at fixed time separation $t_f - t_i$, (ii) over all directions of momenta \mathbf{P} (1) and (iii) over all directions of the ρ meson polarization.

The time dependence $t_f - t_i$ of the correlators $C_{ij}(t_f, t_i)$ (2) contains the information on the energies of the system E_n , and several methods for extracting E_n from C_{ij} are available. We extract two lowest energy levels $E_{n=1,2}$ of the system from the 16×16 correlation matrix $C_{ij}(t_f, t_i)$ using the so called variational method [10], which is the most established among the available methods. Table 1 displays the extracted lowest two energies $E_{n=1,2}$ of the coupled $\pi\pi - \rho$ system for our three choices of total momenta \mathbf{P} (1).

The spectrum E_n in Table 1 for our finite box is the main result of this section. Each energy level corresponds to a different value of $s = E_n^2 - \mathbf{P}^2$, as calculated from E_n and \mathbf{P} in the Table 1. In fact, the table lists values of s obtained from the discrete lattice version of the dispersion relation, which takes into account part of the corrections to $s = E_n^2 - \mathbf{P}^2$ due to finite lattice spacing [6].

3 Extraction of the phase shifts from energy levels

Let us consider the case when the resonance R can strongly decay only to two spinless hadrons H_1 and H_2 , so one has elastic scattering of H_1 and H_2 . We point out that the

\mathbf{P}	level n	$E_n a$	$s a^2$	δ
$\frac{2\pi}{L}(0, 0, 0)$	1	0.5107(40)	0.2608(41)	130.56(1.37)
$\frac{2\pi}{L}(0, 0, 0)$	2	0.9002(101)	0.8103(182)	146.03 (6.58) [*]
$\frac{2\pi}{L}(0, 0, 1)$	1	0.5517(26)	0.1579(29)	3.06 (0.06)
$\frac{2\pi}{L}(0, 0, 1)$	2	0.6845(49)	0.3260(69)	156.41(1.56)
$\frac{2\pi}{L}(1, 1, 0)$	1	0.6933(33)	0.1926(49)	6.87(0.38)
$\frac{2\pi}{L}(1, 1, 0)$	2	0.7868(116)	0.3375(191)	164.25(3.53)

Table 1: The results for two lowest levels $n = 1, 2$ of the coupled $\pi\pi - \rho$ system with three choices of total momentum \mathbf{P} on our lattice with $m_\pi a = 0.1673 \pm 0.0016$, $L = 16a$ and the lattice spacing $a = 0.1239 \pm 0.0013$ fm. The energy levels E_n are obtained by multiplying $E_n a$ with $a^{-1} \simeq 1.6$ GeV. The invariant mass squared of the system is $s = E_n^2 - \mathbf{P}^2$, but the dimensionless value in the table $s a^2$ is obtained using the discretized version of this relation [6].

non-elastic case, when a resonance can decay strongly to several final states (i.e. $H_1 H_2$ and $H_1' H_2'$), is much more challenging for a lattice study.

Suppose one encloses two hadrons $H_1(p_1)$ $H_2(p_2)$ with three-momenta p_1 and p_2 into a large box of size $L \gg$ fm and measures their energy. In a large box, they hardly interact and their energy is equal to sum of individual energies $E^{non-int} = E_{H_1}(p_1) + E_{H_2}(p_2)$ with $E_H(p) = \sqrt{m_H^2 + p^2}$. Now, let's force $H_1(p_1)$ and $H_2(p_2)$ to interact by decreasing the size of the box to L of a few fm. The energy of the system $E(L) = E_{H_1}(p_1) + E_{H_2}(p_2) + \Delta E(L)$ is shifted with respect to $E^{non-int}$: it will increase ($\Delta E(L) > 0$) if the interaction is repulsive and decrease ($\Delta E(L) < 0$) if the interaction is attractive. This simple physical reasoning indicates that the energy shift $\Delta E(L)$ gives info on the interaction.

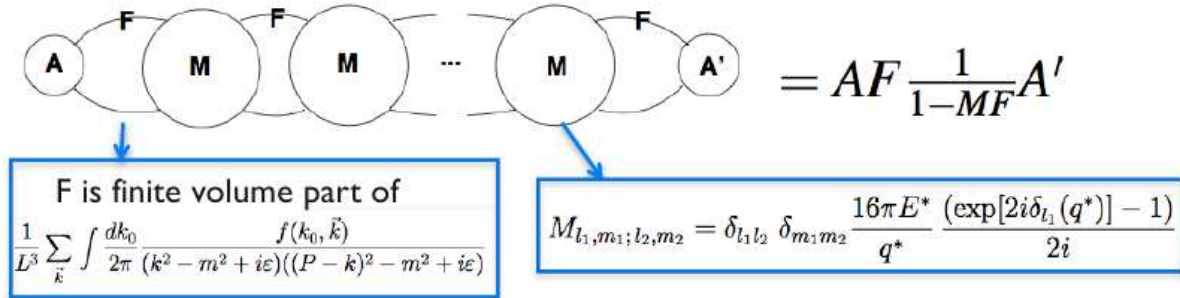


Figure 3: The scattering of two interacting particles as series of the interaction vertex $M(\delta_L)$ and the scattering of non-interacting particles F at finite L [11].

In fact, the energy shift $\Delta E(L)$ and the energy itself $E(L)$ do not only give us "some"

info on the interaction. According to the seminal analytic work of Lüscher [1], $E(L)$ or $\Delta E(L)$ rigorously tells us the value of the elastic scattering phase shift of $H_1 H_2$ scattering at $L \rightarrow \infty$, i.e. $\delta(L = \infty)$:

$$\text{Luscher method :} \quad E(L) \longrightarrow \delta(s, L = \infty) \quad s = E(L)^2 - \mathbf{P}^2 \quad (5)$$

The derivation and the resulting formulae between $E(L)$ and δ are lengthy and rather complicated, but let us briefly explain at least why $E(L)$ contains info on $\delta(L = \infty)$. A nice and clear quantum-field theory derivation is given in [11] and the main message is illustrated in Fig. 3. The scattering of two interacting spin-less hadrons $H_1 H_2$ at finite L (for degenerate case $m_{H_1} = m_{H_2} = m$) is represented in QFT by series of:

- scattering of two non-interacting hadrons at finite L , represented by F . The expression F contains sums over the loop momenta \mathbf{k} , which are allowed in a finite box L with periodic boundary conditions in space. Here $f(k_0, \mathbf{k})$ stands for dependence of the vertices on the left and right on k_0 and \mathbf{k} .
- the interaction vertex M with four hadron legs. This vertex depends on the elastic phase shift δ_l (at infinite volume) for the case of elastic scattering in the l -th partial wave.

The physical scattering requires resummation of the bubbles in Fig. 3, with non-interacting parts F and the interacting parts M , giving $AF \frac{1}{1-MF} A'$. The positions of the poles of the sum $AF \frac{1}{1-MF} A'$ obviously depend on M and therefore on δ_l . The positions of the poles dictate the possible energy levels of the system $E_n(L)$, so the energy levels $E_n(L)$ depend on M and therefore on δ_l .

The purpose of the above illustration was just to indicate why $E_n(L)$ depend on δ_l . In the case of $\pi\pi$ with $J^P = 1^-$, the relevant wave has $l = 1$ and we denote the corresponding phase by $\delta \equiv \delta_1$. The complete analytic relations between $E_n(L)$ and $\delta(s)$ needed for our case of the $\pi\pi$ scattering with $J^{PC} = 1^{--}$ and $I = 1$ are provided in [6] (for every $|\mathbf{P}|$ a different form of relation applies). These allow to extract δ for each of our six energy levels in Table 1 and the resulting phase shifts are given in the same Table.

The presented Lüscher formalism applies only for the case of elastic scattering. The $\pi\pi$ state is the only scattering state in this channel for energies when 4π state cannot be created, i.e., when $s = E_n^2 < (4m_\pi)^2$. For our $m_\pi a = 0.1673$ this is valid for all six levels, with the exception of the level E_1 at $\mathbf{P} = 0$, which is above 4π inelastic threshold. As the Lüscher analysis is not valid above the inelastic threshold, we omit this level from further analysis.

The resulting scattering phase shifts for five values of s are shown in Fig. 4. This is the main result of the lattice study; the resonance properties will be obtained by fitting $\delta(s)$ in the next section.

Note that the resulting phases are determined with a relatively good precision, which is better than in other available lattice studies of ρ at comparable u/d quark masses. The good precision can be traced back to various advanced techniques we used: the distillation method for evaluating contractions, usage of a large interpolator basis and average over all initial time slices, directions of momenta \mathbf{P} and polarizations of ρ .

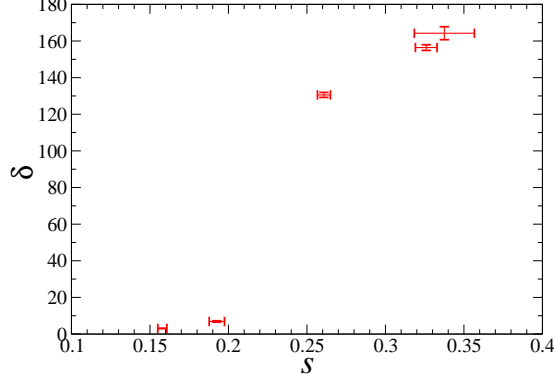


Figure 4: The $\pi\pi$ phase shift $\delta(s)$ (in degrees) for five different values of dimensionless $sa^2 = (E_n a)^2 - (\mathbf{P}a)^2$, extracted from our lattice study [6]. The s is obtained by multiplying sa^2 with $(a^{-1})^2 \simeq (1.6 \text{ GeV})^2$.

4 Extracting resonance mass and width from the phase shift

The phase shift $\delta(s)$ in Fig. 4, obtained directly from the lattice study, can be used to extract the properties of the resonance, in our case the ρ . The phase shift has a typical resonance shape: it passes from $\delta \simeq 0^\circ$ to $\delta \simeq 180^\circ$: the point where it crosses 90° gives the position of the resonance ($s = m_\rho^2$), while the steepness of the rise gives its width Γ_ρ . In particular, δ is related to resonance parameters by expressing the scattering amplitude a_l in terms of δ on one hand, and with Breit-Wigner form in the vicinity of the resonance on the other hand

$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + i\sqrt{s}\Gamma(s)} = \frac{e^{2i\delta(s)} - 1}{2i} . \quad (6)$$

Relation (6) can be conveniently re-written as

$$\sqrt{s}\Gamma(s) \cot \delta(s) = m_\rho^2 - s . \quad (7)$$

The decay width significantly depends on the phase space and therefore on m_π , so the decay width extracted at $m_\pi \simeq 266 \text{ MeV}$ could not be directly compared to the measured width. So, it is customary to extract the $\rho \rightarrow \pi\pi$ coupling $g_{\rho\pi\pi}$ instead of the width, where the width

$$\Gamma(s) = \frac{p^{*3}}{s} \frac{g_{\rho\pi\pi}^2}{6\pi} , \quad \Gamma_\rho = \Gamma(m_\rho^2) \quad (8)$$

depends on the phase space for a P -wave decay and the coupling $g_{\rho\pi\pi}$. The coupling is expected to be only mildly dependent on m_π , which was explicitly confirmed in the lattice studies [5, 7] and analytic study [12]. In (8), p^* denotes the pion momentum in the center-of-momentum frame and we extract it from s using a discretized version of relation $\sqrt{s} =$

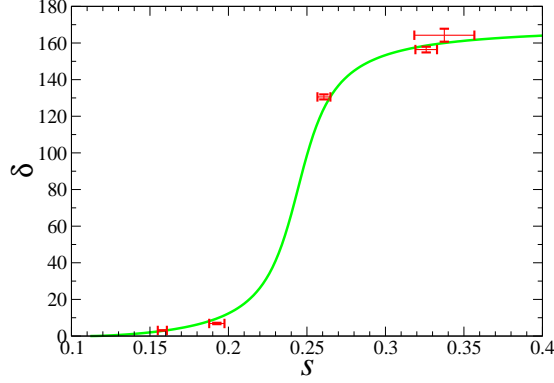


Figure 5: The crosses are the $\pi\pi$ phase shift $\delta(s)$ (in degrees) for five different values of dimensionless $sa^2 = (E_n a)^2 - (\mathbf{P}a)^2$, extracted from our lattice study [6]. The line is the Breit-Wigner fit (7,8) for the resulting m_ρ and $g_{\rho\pi\pi}$ in Table 2. The physical value of s is obtained by multiplying sa^2 with $(a^{-1})^2 \simeq (1.6 \text{ GeV})^2$.

	lattice (this work [6])	exp [PDG]
	$m_\pi \simeq 266 \text{ MeV}$	
m_ρ	$792 \pm 12 \text{ MeV}$	775 MeV
$g_{\rho\pi\pi}$	5.13 ± 0.20	5.97

Table 2: Our lattice results for the resonance parameters [6], compared to the experimental values.

$2\sqrt{m_\pi^2 + p^{*2}}$ [6]. Inserting $\Gamma(s)$ (8) into (7), one obtains an expression for $\delta(s)$ in terms of two unknown parameters: m_ρ and $g_{\rho\pi\pi}$. We fit these two parameters using five values of $\delta(s)$ given in Fig. 4 and Table 1, and we get the values of resonance parameters in Table 2 with small statistical errors.

The resulting ρ -meson mass in Table 2 is slightly higher than in experiment, as expected due to $m_\pi = 266 \text{ MeV} > m_\pi^{\text{exp}}$. The coupling $g_{\rho\pi\pi}$ is rather close to the value $g_{\rho\pi\pi}^{\text{exp}}$ derived from the experimental width Γ_ρ^{exp} .

5 Comparison to other lattice and analytical studies

The comparison of our results for m_ρ and Γ_ρ to two recent lattice studies [5, 7] is compiled in Fig. 8 of [7]. Our result has the smallest error on a given ensemble, demonstrating that accurate lattice determination m_R and Γ_R for (some) resonances is possible now. The other two lattice studies are done for two [7] and four [5] pion masses and explicitly demonstrate mild dependence of $g_{\rho\pi\pi}$ on m_π . The discussion concerning the (dis)agreement of the three lattice studies is given in [7] and will be extended in [13].

The comparison of our $\delta(s)$ to the prediction of the lowest non-trivial order of unitarized Chiral Perturbation Theory [14] is given by the solid line in 6, which has been recalculated for our $m_\pi = 266$ MeV in [15]. The lowest² order prediction does not depend on unknown LECs and agrees reasonably well with our lattice result, given by the bullets.

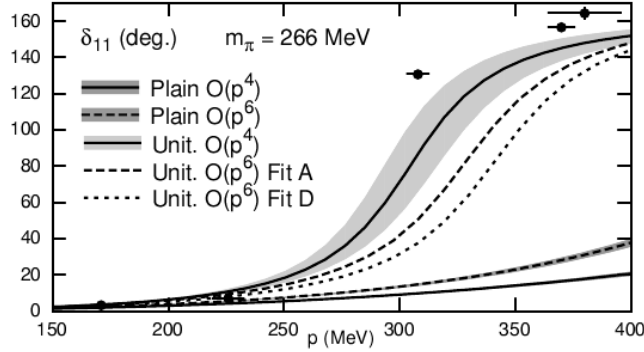


Figure 6: The $\pi\pi$ phase shift in the ρ channel $\delta_{11}(p) \equiv \delta(p^*)$ at $m_\pi = 266$ MeV: the solid line (indicated by “Unit $O(p^4)$ ”) gives prediction of the lowest order of Unitarized Chiral Perturbation Theory [14, 15], while bullets are our lattice data.

6 Conclusions

We highlighted the main physical reasoning, which lies behind the lattice extraction of elastic phase shifts $\delta(s)$ and the resonance parameters m_R and Γ_R . The purpose was to present the general principle of the method and omit the technical details. The method was presented on the example of $\pi\pi \rightarrow \rho \rightarrow \pi\pi$ scattering. This example demonstrates that a proper first-principle treatment of some hadronic resonances on the lattice is now possible.

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²One cannot make a fair comparison between our lattice result and the next-to-lowest order prediction, since it depends on a number of LECs, and some of them have been fixed using m_ρ from another lattice study, which gets a significantly higher m_ρ .

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